P425/1
PURE MATHEMATICS
Paper 1
July/August
3 hours

WAKISSHA JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- *Answer* **all** *the eight questions in section* **A** *and any* **five** *questions from section* **B.**
- *Any additional question(s) answered will* **not** *be marked.*
- *Show* **all** *necessary working clearly.*
- *Begin each answer on a fresh page of paper.*
- *Silent, non programmable scientific calculators and mathematical tables with a list of formulae may be used.*

SECTION A (40 MARKS)

Answer **all** questions in this section.

1. If the equation $x^2 + mx + n = 0$ and $x^2 + px + r = 0$ have common factors, prove that $(n-r)^2 = (m-p)(pn-mr)$. (5 marks)

2. From a class of 14 boys and 10 girls, 10 students are to be elected for a competition in which 5 boys and 5 girls or 2 girls and 8 boys are to go for it. In how many ways can they be selected? (5 marks)

3. Solve $10\sin^2 3x + 10\sin 3x\cos 3x - \cos^2 3x = 2$ for $0^0 \le x \le 120^0$. (5 marks)

4. The area bounded by the curve $x^2 = 4ay$ and the y-axis and lines $y = 0$ and $y = 4b$ is rotated about y-axis through 2π to form a solid. Find the volume of the solid formed. (5 marks)

5. Given the points P(5,4,1) and Q($^-1$, $^-2$, 1). Find the position vector of the point R such that PR: PQ = 2:3. (5 marks)

6. Evaluate $\int x\sin^2 x\cos^2 x\, dx$. (5 marks)

7. Find the equation of the normal to curve $(x-1)^2 + (y+2) = 8$, at the point (3, $^-4$). (5 marks)

8. Q is a variable point given by the parametric equations; $x = \tan\theta - \sin\theta$ and $y = \sin\theta + \tan\theta$. Show that the locus of Q is $(y^2 - x^2)^2 = 16xy$. (5 marks)

SECTION B (60 marks)

Answer any **five** questions from his section.

9. The tangent to the parabola $y^2 = 4ax$ at T $(at^2, 2at)$ meets the x-axis at P. The straight line through T parallel to the axis of the parabola meets the directrix at Q. If S is the focus of the parabola. Prove that TPQS is a rhombus. (12 marks)

10. (a) Mary operates an account with a bank which offers a compound interest of 5% per annum. She opened the account at beginning of 2019 with Shs. 800,000 and continue to deposit the same amount at beginning of every year. How much will she receive at end of 2022 if she made no withdrawal within this period? (6 marks)

(b) Expand $\left(\frac{1+x}{1+3x}\right)^{\frac{1}{3}}$ in ascending powers of x up to the third term, hence by putting $x=\frac{1}{125}$ evaluate cube root of 63 correct to 4 decimal places. (6 marks)

11. Express $f(x)=\frac{3+2x+x^2}{x^3(x+2)}$ into partial fractions. Hence evaluate $\int_2^5 f(x)dx$. (12 marks)

12. (a) A, B and C are non-collinear points with position vectors **a**, **b** and **c** respectively. Point P and Q are on **BC** and **CA** such that **BP**: **PC** = 3:1 and **CQ**: **QA** = 2:3. If point R is on BA produced such that P, Q and R are collinear points. Find in terms of **a**, **b** and **c**, the position vectors of P, Q and R. (8 marks)

(b) Write down the equation of a line which passes through (1, 0, ¯2) in the direction of the vector $\begin{pmatrix}2\\3\\4\end{pmatrix}$ and find the coordinates of the point where the line intersects the plane $4x+3y+2z=25$. (4 marks)

13. (a) Given; $y=be^{-2t}\sin 3t$. Prove that, $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+13y=0$. (6 marks)

(b) A right circular cone is held with it's vertex down beneath a tap leaking at the rate of 2 cm^3s^{-1}. Find the rate of rise of water level when it's depth is 5 cm given that the height and radius of the cone are 15 cm and 5 cm respectively. (6 marks)

14. (a) Prove that $\frac{3\sin\theta+\sin 2\theta}{1+3\cos\theta+\cos 2\theta}=\tan\theta$, hence solve the equation;

$\frac{3\sin\theta+\sin 2\theta}{1+3\cos\theta+\cos 2\theta}=\frac{1}{\cos^2\theta}=2$ for $0^0\leq\theta\leq 360^0$. (6 marks)

(b) Express 7cos A + 24sinA in the form Rsin (A + β) where β is an acute angle and R is a constant. Find the range in which $\frac{2}{7\cos A+24\sin A+10}$ lies. (6 marks)

15. (a) Solve the equation; $2\log_4 x+\log_2(x+6)=6\log_8(x+2)$. (5 marks)

Turn Over

(b) If $Z = \frac{(3-i)(5+12i)}{(1+3i)^2}$

Find the; (i) modulus of Z. (4 marks)

(ii) Argument of Z. (2 marks)

(iii) Polar form of Z. (1 marks)

16. In Focus High School Akiro of 1405 students, all students voted for the head prefect such that the rate of those who had voted is proportional to product of those who had voted and those had not yet voted. 20 students are to supervise the election and they voted before 7.00am, while the other students started to vote at 7:00am. If after 3 hours, 600 students had voted.
Find the;
(i) number of students who had voted after 8:00am. (10 marks)

(ii) time when 800 students had voted. (02 marks)

END

P425/2
APPLIED MATHEMATICS
PAPER 2
July/August
3hours

WAKISSHA JOINT MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

- *Attempt* **all** *questions in section* **A** *and any* **five** *questions from section* **B**.
- *Any additional question(s) answered will* **not** *be marked.*
- **All** *working must be shown clearly.*
- *Begin each answer on a fresh sheet of paper.*
- *Silent non programmable scientific calculators and mathematical tables with a list of formulae may be used.*
- *In numerical work, take* **g** *to be* $9.8ms^{-2}$.
- *State the degree of accuracy at the end of the answer to each question attempted using a calculator or table and indicate* **Cal** *for calculator, or* **Tab** *for mathematical tables.*

SECTION A (40 MARKS)

Answer **all** questions in this section.

1. Events A and B are such that; $P(A \cup B) = \frac{19}{30}$, $P(A) = \frac{5}{15}$ and $P(A/B) = \frac{5}{9}$.

   Determine the;

   Find (a) $P(A \cap B)$ (03 marks)

   (b) $P(\bar{A}/B)$ (02 marks)

2. A particle of weight 10N is suspended by two strings. If the strings make angles of 30^0 and 40^0 to the horizontal, find the tensions in the strings.

   (05 marks)

3. Given that; f (2.09) = 1.9042, f (2.15) = 2.2345, f (2.19) = 2.4979 and f (2.23) = 2.8198. Use linear interpolation or extrapolation to find;

   (a) f(2.11) (03 marks)

   (b) f^{-1} (3.0096) (02 marks)

4. Use the trapezium rule with 6 sub-intervals to estimate.

   $$\int_1^{1.2} x^2 \sin\left(\frac{1}{2}x\right) dx.$$

   Correct to three decimal places. (05 marks)

5. A car approaching a town does two successive half-kilometers in 16 and 20 seconds respectively. Assuming the retardation is uniform, find the further distance the car runs before stopping. (05 marks)

6. A machine manufacturing nails makes approximately 85% that are within the set tolerance limits. If a random sample of 200 nails is taken, find the probability that more than 21 nails will be outside the tolerance limits. (05 marks)

7. The following marks were scored in a mathematics test.

Marks	20-30	30-40	40-45	45-55	55-65	65-75
Frequency density	0.5	1.6	2.4	2.0	1.8	0.6

   Calculate the median. (05 marks)

8. The force, $\underset{\sim}{F}$, acting on a particle of mass 2 kg is given by $\underset{\sim}{F}$ = (5+4t)N, where t is the time in seconds.

   Given that initially the particle is moving at a speed of 5 ms^{-1}, find the speed of the particle when t = 2 seconds. (05 marks)

SECTION B (60 marks)

Attempt any **five** questions from this section.

9. The table below shows the marks scored by students in physics (x) and mathematics (y)

Physics (x)	28	20	40	28	21	22	31	36	29	30	24	21
Mathematics (y)	30	20	40	28	22	25	45	35	27	31	22	33

(a) Draw a scatter diagram to represent the data above. Hence draw the line of best fit. (05 marks)

(b) Use your diagram in (a) to estimate the score in Physics when the score in Mathematics is 24. (01 mark)

(c) Calculate the rank correction coefficient for the data and comment on your result at 5% level significance. (06 marks)

10. A rectangle ABCD (3m x 4m) has forces of magnitudes 5N, 10N, 15N, 20N and 15N acting along the lines BA, CB, DC, AD and CA respectively. If $\overline{AB} = 3$m is the positive x-axis and $\overline{AD} = 4$m is the positive y-axis; find the;

(a) magnitude of the resultant force and its direction. (08 marks)

(b) line of action of the resultant and where it cuts the x-axis. (04 marks)

11. A random variable x has a probability density function given by;

$$f(x) \begin{cases} \lambda x & ; 0 \leq x \leq 1 \\ \frac{\lambda}{2}(3-x); & 1 \leq x \leq 3 \\ 0 & ; \text{otherwise} \end{cases}$$

Where λ is a constant.

Determine the;

(a) value of λ. (03 marks)

(b) expected value of x. (02 marks)

(c) variance of x. (02 marks)

(d) cumulative distribution function, f(x) and hence $P(0.5 \leq x \leq 2.5)$ (05 marks)

12. Two cyclists P and Q are 11 km apart with Q on a bearing of 110^0 from P. Cyclist P is riding at 5 kmh^{-1} due North-East and Q is riding due $N15^0W$ at 8 kmh^{-1}.

Find the;

(a) closest distance between them in the subsequent motion. (09 marks)

(b) time that elapses before they are closest to each other. (03 marks)

13. (a) The numbers $x = 3.7$ and $y = 70$ are each rounded off with percentage error of 0.2 and 0.05 respectively. While the number $z = 26.23$ is calculated with relative error of 0.04. Find the interval within which the exact value of $\frac{x}{y-z}$ lies; correct to 4 significant figures. (06 marks)

Turn Over

(b) The height and radius of a cylindrical water tank are given as H = 3.5 ± 0.2 and R = 1.4 $\pm$ 0.1 respectively. Determine in m^3, the least and greatest amount of water the tank can contain. Hence, calculate the maximum possible error in your calculation. (06 marks)

14. Given the equation $xe^{-x} - 3x + 4 = 0$

(a) (i) Show that the equation has a root between $x = 1$ and $x = 3$. (03 marks)

(ii) Use linear interpolation to obtain an approximation of the root to two decimal places. (02 marks)

(b) Use the Newton Raphson formula to find the root of the equation by performing two interactions correct to three decimal places. (07 marks)

15. A car of mass 1,200 kg pulls a trailer of mass 300 kg up a slope of 1 in 100 against a constant resistance of 0.2N per kg. Given that the car moved at a constant speed of 1.5 ms^{-1} for 5 minutes, calculate the;

(a) tension in the tow bar. (05 marks)

(b) work done by the engine of the car during this time. (04 marks)

(c) total resistance if the engine developed power of 15 kW at a maximum speed of 120 kmh^{-1} on a level road. (03 marks)

16. The speeds of cars passing a certain point on a motor way can be taken to be normally distributed. Observations along the motor way at a certain point show that 95% of the cars are travelling at less than 85 kmh^{-1}while 10% of the cars are travelling at less than 55 kmh^{-1}.

(a) Determine the average and standard deviation of the speeds of the cars passing that point along the motor way. (06 marks)

(b) If a random sample of 25 cars is selected, find the;

(i) Probability that their average speed is not more than 70 kmh^{-1}. (03 marks)

(ii) 95% confidence interval for the average speed. (03 marks)

END

WAKISSHA JOINT MOCK EXAMINATIONS
MARKING GUIDE
Uganda Advanced Certificate of Education
UACE August
Mathematics P425/2

Mr Makumba Gerald

1. (a) $P(A/B) = \frac{P(AnB)}{P(B)}$

$\frac{5}{9} = \frac{P(AnB)}{P(B)} \Rightarrow P(AnB) = \frac{5}{9}P(B)$

$P(AUB) = P(A) + P(B) - (P(AnB)$ ~~A1~~ (M1) for substitution

$P(B) = \frac{27}{40}$ ~~B1~~ A1 (for P(B))

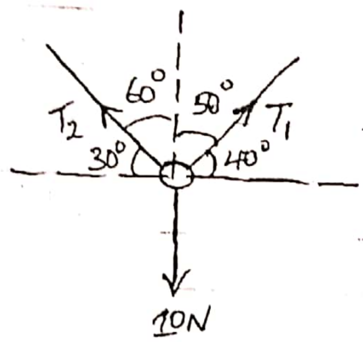
$P(AnB) = \frac{5}{9} \times \frac{27}{40} = \frac{3}{8}$ A1

(b) $P(\bar{A}/B) = \frac{P(\bar{A}nB)}{P(B)} = \frac{P(B) - P(AnB)}{P(B)}$

$= \frac{\frac{27}{40} - \frac{3}{8}}{27/40}$ M1 (for sub.n)

$= \frac{4}{9}$ A1

2.



B1 (If a candidate uses a straight edge)

$\frac{T1}{\sin 120^0} = \frac{10}{\sin 110^0} = \frac{T2}{\sin 130^0}$ M1

$\frac{T1}{\sin 120^0} = \frac{10}{\sin 110^0} \Rightarrow T1 = \frac{10}{\sin 110^0} \times \sin 120^0$ ~~A1~~

$= 9.21605N$ } at least 2 dp. A1

$T2 = \frac{10}{\sin 110^0} \times \sin 130^0$

$= 8.15207N$ at least 2 dp. A1

3. (a)

x	2.09	2.11	2.15
$f(x)$	1.9042	m	2.2345

B1

$$\frac{2.15-2.09}{2.11-2.09}=\frac{2.2345-1.9042}{m-1.9042}$$

M1

$= 2.0143$

A1

03

(b)

x	2.19	2.23	~~N~~ n
f(x)	2.4979	2.8198	3.0096

$$\frac{n-2.19}{2.23-2.10}=\frac{3.0096-2.4979}{2.8198-2.4979}$$

M1

$n = 2.2536$

$\therefore f(3.0096) = 2.25$

A1

02

4. Let $y = x^2\sin\left(\frac{1}{2}x\right)$

$h= \frac{1.2-1}{6}$ B1 $= \frac{1}{30}$ → B1

x	**y**	
1	0.4794	
$\frac{31}{30}$		0.5275
$\frac{16}{15}$		0.5785
$\frac{11}{10}$		0.6325
$\frac{17}{15}$		0.6895
$\frac{7}{6}$		0.7497
1.2	0.8131	
sum	1.2925	3.1777

4dps

~~B1 for all x-values~~ B1 values of x

~~B1 for all values of y~~ B1 values of y

B1 all y values

B1 summations

$$\int_1^{1.2} x^2\sin\left(\frac{1}{2}\right)dx \cong \frac{1}{2}\times\frac{1}{30}(1.2925+2\times 3.1777)$$

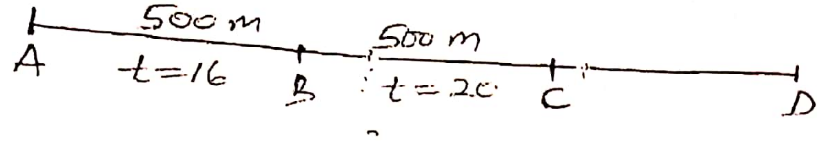
M1

$\cong 0.127465$ — should be seen before A1

$\cong 0.127$ → Award A1 if we see the value above.

A1

5.



$S = ut + \frac{1}{2}at^2$

$AB; 500 = 16u + \frac{1}{2}\times a\times(16)^2 \Rightarrow 31.25 = u + 8a --- (i)$

$AC; 1000 = 36u + \frac{1}{2}\times a\times(36)^2 \Rightarrow \frac{250}{9} = u + 18a --- (ii)$ B1 for the two Equations ~~M1~~

$equ(ii) - eqn(i); 10a\frac{-125}{36} \Rightarrow a = -0.3472ms^{-2}$ (4 dps) A1

$u = 31.25 - 8\times(-0.3472) = 34.0278^{ms^{-1}}$ A1

$LetAD = S; u\sin gV^2 = u^2 + 2as$ on Subsn

$0^2 = (34.0278)^2 + 2x - 0.3472\times s$ M1

$S = 1667.4678m$

$Extradistance = 1.667.4678 - 1000 = 667.4678m$ A1 At least 2 dps.

6. P = 0.15 q = 0.85 n = 200

$\mu = np = 200 \times 0.15 = 30$ B1

$\sigma = \sqrt{30\times0.85} = 5.04975$ B1

Let x be a r.v for nails ~~outside the tolerance limits~~.

$P(x > 21) = P\left(z > \frac{21.5-30}{5.04975}\right)$ M1

$= P(2 > -1.68325)$

$= 0.5 + P(0czc1.683)$

$= 0.5 + 0.45381$ M1

$= 0.95381$ A1

Iff c(i) not shown. Give B1B1

7.

marks	B1	f.d	f B1	c.f (F) B1
20-30	10	0.5	5	5
30-40	10	1.6	16	21
40-45	5	2.4	12	33
45-55	10	2.0	~~10~~ 20	~~43~~ 53
55-65	10	1.8	~~9~~ 18	~~52~~ 71
65-75	10	0.6	~~3~~ 6	~~55~~ 77

Position of median class =

$\left(\frac{\cancel{55}\ 77}{2}\right)^{th} \cancel{=27.5^{th}}$ 38.5th

$Median = 45 + \left(\frac{38.5-33}{20}\right)\times10$ M1

~~Median=40+$\left(\frac{27.5-21}{12}\right)(5)$ B1M1~~

~~=27.7083 A1~~

$= 47.75$ A1

8.

$$\underset{\sim}{a} = \frac{F}{m} = \frac{5}{2} + \frac{4}{2} = (2.5 + 2t)ms^{-2} \quad \cancel{B1} \text{ B1}$$

$$\underset{\sim}{v} = \int a dt = \int (2.5+2t)dt$$

$$\underset{\sim}{v} = 2.5t + t^2 + C \quad \cancel{M1} \text{ M1}$$

$t = 0$, $V = 5ms^{-1}$

$\Rightarrow 5=2.5(0)+(0)+C \Rightarrow C=5$ ~~B1~~ B1 — Expression for V.

$V = 2.5t + t^2 + 5$ B1

$t=2, V=2.5+2^2+5$ M1 → Subs.

$=14ms^{-1}$ A1 → Accuracy

9. (a) Refers to graph paper

(b) ~~23.5 B1~~ 23–25 B1

(c)

Rp	Pm	$d^2 = (Rp - Rm)^2$
6.5	6	0.25
12	12	0
1	2	1
6.5	7	0.25
10.5	10.5	0
9	9	0
3	1	4
2	3	1
5	8	9
4	5	1
8	10.5	6.25
10.5	4	42.25
B1	B1	$\Sigma d^2=65$
		B1

Rp	Rm	d	d^2
6.5	7	-0.5	0.25
		a	a
1	12		
12	11	1	1
6.5	6	0.5	0.25
2.5	2.5	0	0
4	4	0	0
10	12	2	4
11	10	1	1
8	5	3	9
9	8	1	1
5	2.5	2.5	6.25
2.5	9	-6.5	42.25 B1
B1	B1	B1	$\Sigma d^2 = 65$

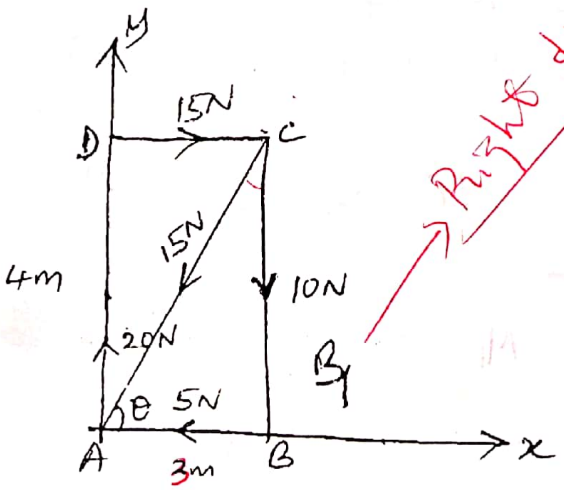
$$\rho = 1 - \frac{6 \times 65}{12(12^2 - 1)} \quad \text{M1}$$

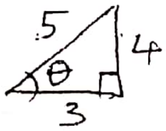
$$= 0.77 \quad \text{A1}$$

B1

sence0.77>0.58then it's significant at 5%level

10. (a)





$\sin\theta = \frac{4}{5}$

$\cos\theta = 3/5$ B1

— θ should be on the diagram

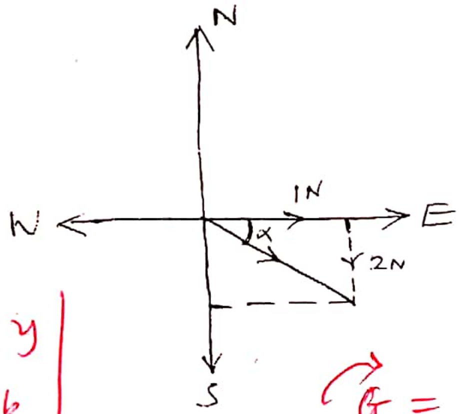
$$\underset{\sim}{F}=\begin{pmatrix}-5\\0\end{pmatrix}+\begin{pmatrix}15\\0\end{pmatrix}+\begin{pmatrix}-15 & \cos\theta\\-15 & \sin\theta\end{pmatrix}+\begin{pmatrix}0\\20\end{pmatrix}+\begin{pmatrix}0\\-10\end{pmatrix}$$ ~~B1N1~~ M1 ⇒ $\theta = 53.13^0$

⟶ for the Resolutes

$$=\begin{pmatrix}-5\\0\end{pmatrix}+\begin{pmatrix}15\\0\end{pmatrix}+\begin{pmatrix}-15\text{ x } \frac{3}{5}\\-15\text{x } \frac{4}{5}\end{pmatrix}+\begin{pmatrix}0\\15\end{pmatrix}$$ ~~M1~~ M1 (M1 If didn't find Cos θ & Sin θ)

$$=\begin{pmatrix}1\\-2\end{pmatrix}$$ A1

$|\underset{\sim}{F}| = \sqrt{1^2+(-2)^2}=\sqrt{5}=2.2361\text{N}$ A1



$$\tan\ \alpha = \frac{2}{1}$$

$$\alpha = \tan^{-1}(2) = 63.43^0$$ B1

$$Direction = S26.57^0 E$$ B1

$G = \begin{vmatrix} x & y \\ a & b \end{vmatrix}$

Σx Σy

$\begin{pmatrix} a & x \\ b & y \end{pmatrix}$

$\vec{G} = \begin{vmatrix} x & y \\ 1 & -2 \end{vmatrix}$ ⇒ $-90 = -2x - y$.

(b) A; G=-10x3 + -15x4 = -90Nm M1 ⟶ Taking moments about A

-90 - x(-2) + y(1) = 0 ⇒ 2$\cancel{x}$ + y - 90 = 0 A1 ⟶ Forming an Equation.

Resultant cuts x-axis when y=0

⇒ 2x = $^+$90 ; $\cancel{x}$ = 45 ~~M1~~ M1 ⟶ for putting in y=0.

The point is (45, 0) A1

$\boxed{\text{OR 45 m from A}}$ or $\boxed{\text{42 from B}}$

04

11. (a) $\int_0^1 \lambda x dx+\int_1^3 \frac{\lambda}{2}(3-x)dx=1$

$\frac{\lambda x^2}{2}\Big|_0^1+\frac{\lambda}{2}(3x-\frac{x^2}{2})\Big|_1^3=1$ M1

$\left(\frac{\lambda}{2}-0\right)+\frac{\lambda}{2}\left\{\left(9-\frac{9}{2}\right)-\left(3-\frac{1}{2}\right)\right\}=1$ B1

$\frac{\lambda}{2}\left\{\frac{2+9-5}{2}\right\}=1$

$\lambda=\frac{2}{3}$ A1

03

(b)

$$E(X)=\frac{2}{3}\int_0^1 x^2dx+\frac{1}{3}\int_1^3\left(3x-x^2\right)dx$$

$$=\frac{2}{9}x^3\Big|_0^1+\frac{1}{3}\left(\frac{3}{2}x^2-\frac{x^3}{3}\right)\Bigg|_1^3 \quad \text{M1}$$

$$=\frac{2}{9}(1\text{-}0)+\frac{1}{3}\left\{\left(\frac{27}{2}\text{-}9\right)\text{-}\left(\frac{3}{2}\right)\text{-}\left(\frac{1}{3}\right)\right\} \quad \text{B1}$$

$$=\frac{4}{3} \quad \text{A1}$$

02

(c)

$$E(x^2)=\frac{2}{3}\left[\frac{x4}{4}\right]_0^1+\frac{1}{3}\left[x^3\text{-}\frac{x^4}{4}\right]\Bigg|_1^3 \quad \text{M1}$$

$$=\frac{2}{3}\times\frac{1}{4}+\frac{1}{3}\left[\left(27\text{-}\frac{81}{4}\right)\text{-}\left(1\text{-}\frac{1}{4}\right)\right]$$

$$=\frac{1}{6}+2$$

$$=\frac{13}{6}$$

$$\text{var}(x)=\frac{13}{6}\text{-}\left(\frac{4}{3}\right)^2=\frac{7}{18}$$

A1

02

(d) $$F(x)=\int_0^x\frac{2}{3}xdx=\left[\frac{1}{3}x^2\right]_0^x=\frac{x^2}{3} \quad \text{B1}$$

$$F(1)=\frac{1}{3}$$

$$F(x)=\int_1^x\frac{1}{3}(3-x)dx+\frac{1}{3}$$

$$=\frac{1}{3}\left(3x-\frac{x^2}{2}\right)\Bigg|_1^x+\frac{1}{3}$$

$$=\frac{1}{3}\left\{\left(3x-\frac{x^2}{2}\right)-\left(3-\frac{5}{2}\right)\right\}+\frac{1}{3} \quad \text{B1}$$

$$=\frac{1}{3}\left(3x-\frac{x^2}{2}-\frac{5}{2}\right)+\frac{1}{3}$$

$$F(x)=x-\frac{x^2}{6}-\frac{1}{2}$$

$$F(3) = 3 - \frac{9}{6} - \frac{1}{2} = 1$$

$$F(x) = \begin{cases} 0 & x \le 0 \\ \dfrac{x^2}{3} & 0 \le x \le 1 \\ x - \dfrac{x^2}{6} - \dfrac{1}{2} & 1 \le x \le 3 \\ 1 & x \ge 3 \end{cases}$$ B1

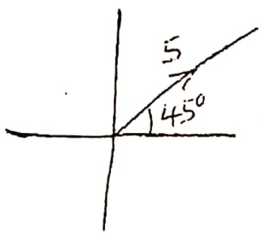
$$P(0.5 \le \times 2.5) = F(2.5) - F(0.5)$$

$$= \left(2.5 - \frac{(25)^2}{6} - \frac{1}{2}\right) - \frac{(0.5)^2}{3}$$ M1

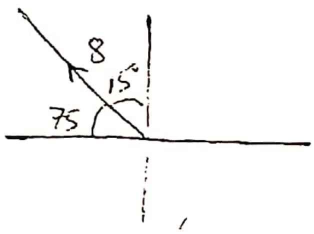
$= 0.8750$ A1

05

12. (a)



$$\underset{\sim}{v}p = \begin{pmatrix} 5\cos 45^0 \\ 5\sin 45^0 \end{pmatrix} = \begin{pmatrix} 3.5355 \\ 3.5355 \end{pmatrix}$$ B1 ✓



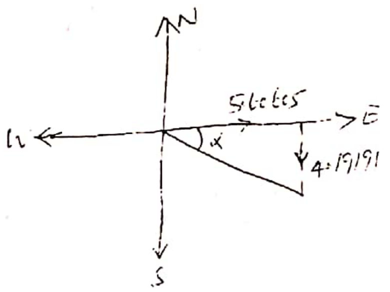
$$\underset{\sim}{v}p = \begin{pmatrix} -8\cos 75^0 \\ 8\sin 75^0 \end{pmatrix} = \begin{pmatrix} -2.07055 \\ 7.72741 \end{pmatrix}$$ B1 ✓

$$p\underset{\sim}{v}Q = \begin{pmatrix} 3.5355 \\ 3.5355 \end{pmatrix} - \begin{pmatrix} -2.07055 \\ 3.5355 \end{pmatrix} = \begin{pmatrix} +5.60605 \\ -4.19191 \end{pmatrix}$$ ✓ B1

~~M1~~

$$\left|p\underset{\sim}{v}Q\right| = \sqrt{(5.60605)^2 + (-4.19191)^2}$$ M1

$= 7kmh^{-1}$ A1 ✓

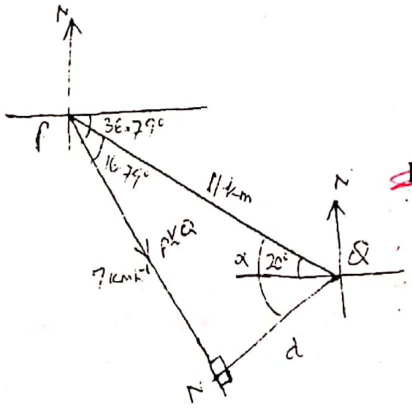


$$\propto = \tan^{-1}\left(\frac{4.19191}{5.60605}\right)$$

$= 36.79^0$ B1

~~Direction is S36.79^{0}E~~ (Not required)

~~A1~~



~~B1~~

B1

$pV_Q \cdot pr_Q = 0.$

M1

(b) $\sin 16.79 = \frac{d}{11} \Rightarrow d = 11\sin 16.79^0 = 3.1775\text{km}$ ~~A1~~ A1

(b) $\cos 16.79^0 = \frac{PN}{11} \Rightarrow \overline{PN} = 10.531069$ B1 ~~B1~~

$\text{time} = \frac{10.531069}{7} = 1.5044$ hours A1 ~~A1~~

~~M1~~ M1

13. (a)

$e_x = \frac{0.2}{100} \times 3.7 = 0.0074$ B1

$e_y = \frac{0.05}{100} \times 70 = 0.035$ B1

$e_z = 0.04 \times 26.23 = 1.0492$ B1

$\text{let} P = \frac{x}{y\text{-}z} = \frac{3.7}{70\text{-}26.23}$

$\text{Pmax} = \frac{3.7+0.0074}{(70\text{-}0.035)\text{-}(26.23+1.0492)}$ B1

$\cong 0.08685$

$\text{Pmin} = \frac{3.7\text{-}0.0074}{(70+0.0035)\text{-}(26.23\text{-}1.0492)}$ M1

$=0.0823824$

$\cong 0.08238$

Interval=$[0.08238, 0.08685]$ A1

06

(b) $\pi R^2 H$

M1 M1

$\max = \pi\,(3.5 + 0.2)^2(1.4 + 0.1) = \cancel{64.5126}$ ~~B1~~ 26.1526 B1

M1 M1

$\min = \pi\,(3.5\text{-}0.2)^2(1.4 - 0.1) = \cancel{44.4755}$ ~~B1~~ 17.5277 B1

Max. possible error $= \frac{1}{2}(\cancel{64.512644.4755})$ M1 $= \frac{1}{2}(26.1526 - 17.5277)$ M1

$= \cancel{10.0185}$ A1 $= 4.3245$ A1 **06**

14. (a) (i) $f(x) = xe^{-x} - 3x + 4$

$f(1) = e^{-1} - 3 + 4$

$= 1.3679$ ⟶ B1

$f(3) = 3e^{(-3)} - 9 + 4$

$= \cancel{-4.9502}$ -4.8306 B1

$f(i).f(3) = (1.3679)(-4.9502) < 0$ ~~B1~~ ⟶ B1

since $f(i).f(3) < 0$, then the root lies between 1 and 3

03

(ii)

x	1	x_0	3
F(x)	1.3679	0	~~-4.9512~~ -4.8506

$$\frac{3-1}{\cancel{-4.9502}-1.3679} = \frac{x_0-1}{0-1.3679} \quad \text{M1}$$

-4.8506-

$x_0 = 1.43\cancel{3009924}$

~~$x_0 \cong 1.43$~~ A1 $X_0 = 1.44$ A1

02

(b) $f^1(x) = -xe^{-x} + e^{-x} - 3$ ~~M1B1~~ B1

$$Xn+1 = xn - \left(\frac{x_n e^{-xn} - 3xn + 4}{-x_n e^{-xn} + e^{-xn} - 3}\right) \quad \text{~~M1B1~~ B1}$$

~~written as~~ $X_{n+1} = \dfrac{-x_n^2 e^{-x_n} - 4}{e^{-x_n} - x_n e^{-x_n} - 3}$ ~~Formula~~

$$x_1 = 1.43 - \left(\frac{1.43e^{-1.43} - 3(1.43) + 4}{-1.43e^{-1.43} + e^{-1.43} - 3}\right) \quad \text{~~B1~~ B1}$$

$= 1.4468$ B1

$|x_1 - x_0| = |1.4468 - 1.43| = 0.0168 > 0.0005$ (Ignore) Not needed.

$$x_2 = 1.4468 - \left(\frac{1.4468e^{-1.4468} - 3(1.4468) + 4}{-1.4468e^{-1.4468} + e^{-1.4468} - 3}\right) \quad \text{~~B1~~ B1}$$

$= 1.4468.$ B1

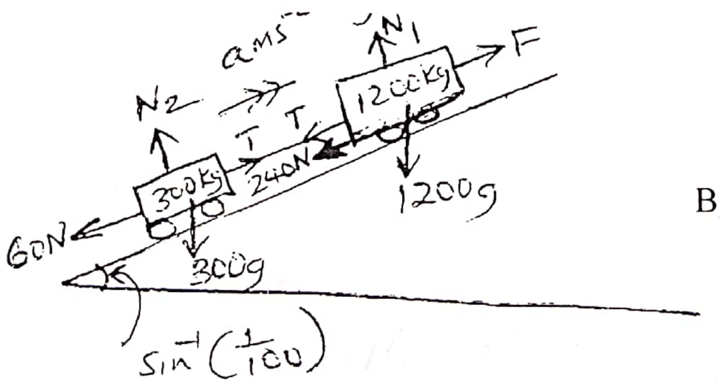
$(x_2 - x_1) = |1.4468 - 1.4468| = 0 < 0.0005$ ~~B1~~ Not needed.

$x = 1.4468$

$\therefore x = 1.447$ A1

07

15. (a) Resistance of the car = 0.2 x 1200 = 240N = R_1
Resistance of the trailer = 0.2 x 300 = 60N R_2



B1

$F = 240 + 1200g\sin\theta + 60 + 300g\sin\theta$

$= 240 + 1200g \times 9.8 \times \frac{1}{100} + 60 + 300 \times 9.8 \times \frac{1}{100}$ M1

$= 447N$ A1

$F = T + 240 + 1200\, g \sin\theta$

$447 = T + 240 + 1200 \times 9.8 \times \frac{1}{100}$ M1

$\Rightarrow T = 89.4N$ A1

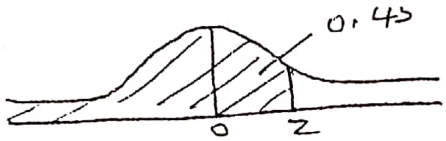
(b) Work done = force x distance

But $S = ut = 1.5 \times 5 \times \overset{M_1}{60} = 450m$ B_1

Work done $= 447 \times \overset{M_1}{450} = 201,150\ J$ A_1

(c) Speed $= \frac{120 \times 1000}{3600} = \frac{100}{3} ms^{-1}$ B_1

$R_1 + R_2 = \frac{15000 \times 3}{100} = 450N$ A_1 M_1



16. (a) $P(X<85) = 0.95$

$P(Z< \frac{85-\mu}{\sigma}) = 0.95$

$\frac{85-\mu}{\sigma} = 1.645 \Rightarrow 85-\mu = 1.645\,\sigma$ ---------- (i) ~~B1~~ B1

$P(X < 55) = 0.1$

$P(Z< \frac{55-\mu}{\sigma}) = 0.1$

$\frac{55-\mu}{\sigma} = -1.282 \Rightarrow 55-\mu = -1.282\sigma$ ---------- (ii) ~~B1~~ B1

M1 — solving simultaneous eqns

M1 Eqn (i) – eqn(ii); $30 = 2.927\ \sigma = 10.2494$ ~~A1~~ A1

$\therefore \mu = 85 - 1.645 \times 10.2494$ ~~M1~~ M1

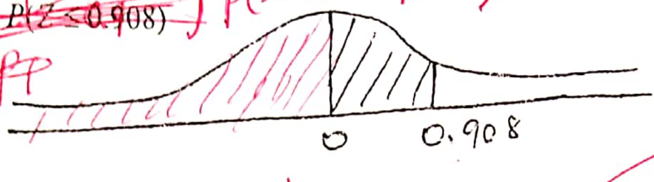
68.1397

= ~~68.1373~~. A1 A1

06

(b) (i) $P(\overline{X} \le 70) = P\left(Z \le \dfrac{70-68.13973}{\left(\dfrac{10.2494}{5}\right)}\right)$ M₁

~~$= P(Z \le 0.908)$~~ $P(Z \le 0.908)$



$= 0.5 + 0.31793$ M1 M1

~~$= 0.18207.$~~ A₁

$= 0.8179$ A1

03

03

(ii) $Z_{0.475} = 1.96$

Upper limit of X = 68.13973 + 1.96 x 10.2494 M₁

= 88.2286

Lower limit of X = 68.13973 − 1.96 x 10.2494 M₁

= 48.0509

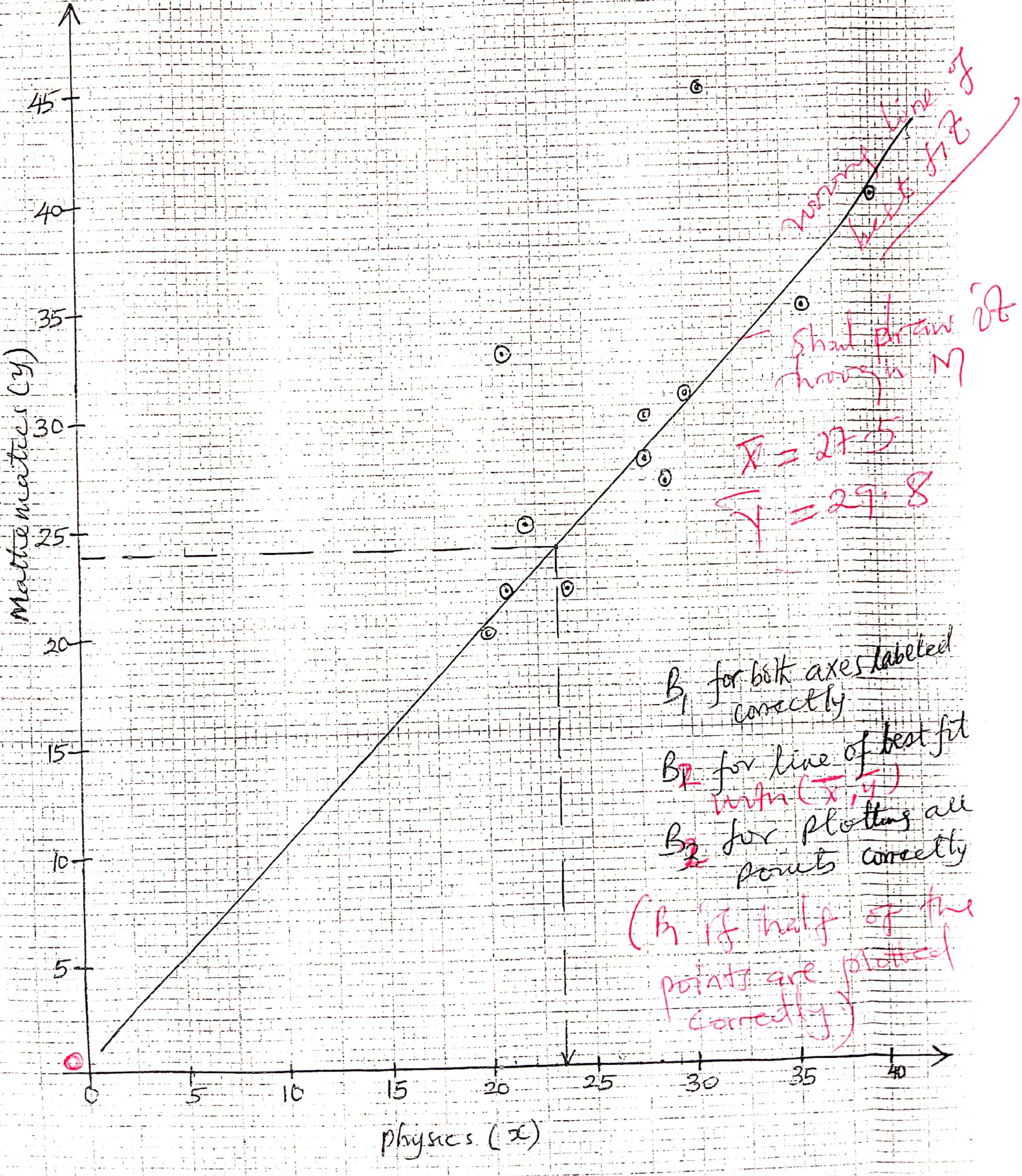
Interval = $[48.0509,\ 88.2286]$ A₁

03

END

9 (a)

A scatter diagram for mathematics (y) against Physics (x



15

WAKISSHA JOINT MOCK EXAMINATIONS
MARKING GUIDE
Uganda Advanced Certificate of Education
UACE August
MATHEMATICS P425/1

Q1	Let common factor be $x-\alpha \Rightarrow x=\alpha$ is common root $\Rightarrow \alpha^2+m\alpha+n=0$ ________ (i) $\alpha^2+p\alpha+r=0$ ________ (ii) (i) -(ii) : $\alpha^2+m\alpha+n=0$ $\alpha^2+p\alpha+r=0$ $(m-p)\alpha+(n-r)=0$ $\alpha=\frac{-(n-r)}{m-p}$ ______ * $p\alpha^2+pm\alpha+pn=0$ P(i) – m(ii) $m\alpha^2+pm\alpha+mr=0$ $(p-m)\alpha^2+pn-mr=0$ $(m-p)\alpha^2=pn-mr$ ** Substitute for α^2 in ** using * $(m-p)\left(\frac{-n-r}{m-p}\right)^2=pn-mr$ $\therefore$ $(n-r)^2=(m-p)(pn-mr)$	M_1 for two equations got substituting the root. B_1 for α a subject B_1 for the equation (xx) M_1 for substituting α^2 A_1 for required solution. **05**
Q2	Selection made ${}^{14}C_5$ and ${}^{10}C_5$ OR ${}^{14}C_8$ and ${}^{10}C_2$ $= {}^{14}C_5\times{}^{10}C_5+{}^{14}C_8\times{}^{10}C_2$ = (2002) (252) +(3003) (45) = 504504 + 135135 = 639639 ways	M_1 for showing how they are combined.. M_1 for ~~replacing and, or~~ adding combination correctly. M_1 for correct values of combinations. M_1 for addition A_1 for correct number of ways **05**
Q3	$\frac{10\sin^2 3x}{\cos^2 3x}+\frac{10\sin 3x\cos 3x}{\cos^2 3x}-\frac{\cos^2 3x}{\cos^2 3x}=\frac{2}{\cos^2 3x}$ $10\tan^2_{3x}+10\tan 3x-1=2(1+\tan^2 3x)$ $8\tan^2_{3x}+10\tan 3x-3=0$ Let $\tan 3x$ be t $\Rightarrow 8t^2+10t-3=0$ $t=\frac{-10\pm\sqrt{10^2--4\times 8\times --3}}{2\times 8}$ = 0.25 or -1.5	M_1 for dividing through by $\cos^2 3x$ or $\sin^2 3x$. M_1 for method of solving quadratic equation.

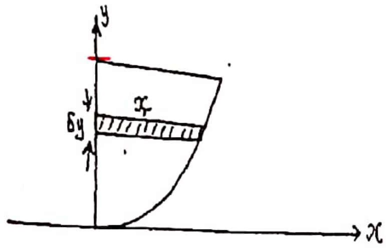
<table>
<tr><td></td><td>$\Rightarrow \tan 3x = 0.25 \text{ or } \tan 3x = -1.5$

$3x = \tan^{-1}(0.25) \text{ or } 3x = \tan^{-1}(-1.5)$
$0^0 \le 3x \le 360^0$
$= 14.04^0$ and 194.04^0 or 123.96^0 and 303.69^0
$\therefore x = 4.68^0, 41.23^0, 64.68^0$ and 101.23^0</td><td>A1

M1

A1</td><td>A_1 for the two values of $\tan 3x$
M_1 for reading from tan table within correct range.
A_1 for the four correct angles.
05 final answer</td></tr>
<tr><td>Q4</td><td>

y	0	2b	4b
x	0	$\pm 2\sqrt{(2ab)}$	$\pm 4\sqrt{ab}$



Required volume $= \pi \int_0^{4b} (4ay)dy$

$= \pi\left[2ay^2\right]_0^{4b}$

$= \pi(32ab^2 - 0)$

$= 32ab^2\pi$ cubic units</td><td></td><td>B_1 for showing the area or for getting correct limits.

M_1 for integrating

A_1 for correct integral

M_1 for substituting limits.

A_1 for required volume.
05</td></tr>
<tr><td>Q5</td><td>From PR : PQ = 2:3.
$3\underset{\sim}{PR} = 2\underset{\sim}{PQ}$
$3(\underset{\sim}{OR} - \underset{\sim}{OP}) = 2(\underset{\sim}{OQ} - \underset{\sim}{OP})$
$\underset{\sim}{OR} = \frac{1}{3}(2OQ + \underset{\sim}{OP})$
$= \frac{1}{3}\left[2\begin{pmatrix}-1\\-2\\1\end{pmatrix} + \begin{pmatrix}5\\4\\1\end{pmatrix}\right]$
$= \frac{1}{3}\begin{pmatrix}3\\0\\3\end{pmatrix}$
$\therefore \underset{\sim}{OR} = \begin{pmatrix}1\\0\\1\end{pmatrix}$</td><td></td><td>$B_1$ for equating PR and PQ.

M_1 for making OR a subject.

M_1 for substituting position vectors P and Q.

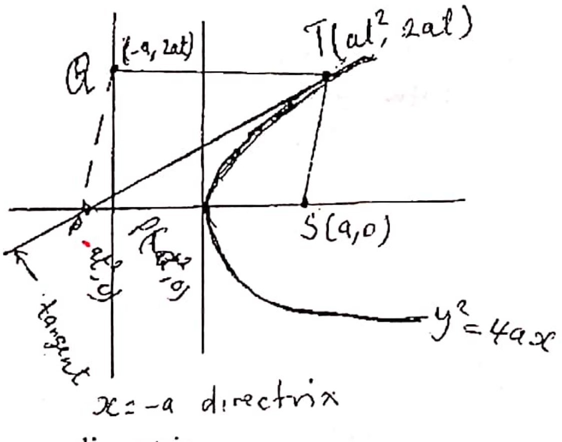
M_1 for simplifying.

A_1 for correct position vector of point R.
05</td></tr>
<tr><td>Q6</td><td>$\int_0^{\frac{\pi}{2}} x\sin^2 x\cos^2 x dx$

$\int_0^{\frac{\pi}{2}} x(\sin x\cos x)^2 dx$

$\int_0^{\frac{\pi}{2}} x\left(\frac{1}{2}\sin 2x\right)^2 dx$</td><td></td><td></td></tr>
</table>

	$\frac{1}{4}\int_0^{\frac{\pi}{2}} x\sin^2 2x.dx$ M1 $\frac{1}{4}\int_0^{\frac{\pi}{2}} x\left[\frac{1}{2}(1-\cos 4x)\right]dx$ $\frac{1}{8}\int_0^{\frac{\pi}{2}} x(1-\cos 4x)dx$ $\frac{1}{8}\int_0^{\frac{\pi}{2}} xdx-\frac{1}{4}\int x\cos 4xdx$ A1 $\frac{1}{8}\left[\frac{x^2}{2}\right]_0^{\frac{\pi}{2}}-\frac{1}{4}x\frac{\sin 4x}{4}-\int\frac{1}{4}\sin 4xdx$ $\frac{1}{8}\frac{\pi^2}{8}-\frac{1}{32}[x\sin 4x]_0^{\frac{\pi}{2}}+\frac{1}{32}\int_0^{\frac{\pi}{2}}\sin 4xdx$ $\frac{\pi^2}{64}-\frac{\pi}{64}\sin 2\pi-0-\frac{1}{64}[\cos 4x]_0^{\frac{\pi}{2}}$ 128 $\frac{\pi^2}{32}-0-0-\frac{1}{64}(\cos 2\pi-\cos 0)$ $=\frac{\pi^2}{32}$ $\frac{\pi^2}{64}$. A1	If not provided with limits. B_1 for correct form which can be integrated. M_1 for all integration. A_1 for all correct integral. M_1 for all substitution of limits. A_1 correct value. **05**
Q7	$\frac{d}{dx}\left((x-1)^2\right)+\frac{d}{dx}\left((y+2)^2\right)=\frac{d}{dx}(8)$ $2(x-1)+2(y+2)\frac{dy}{d}=0$ OR. $\frac{d}{dx}(x-1)^2+\frac{d}{dx}(y+2)=\frac{d}{dx}(8)$ $\frac{dy}{dx}=\frac{-2(x-1)}{2(y+2)}=\frac{^-(x-1)}{y+2}$ $\frac{dy}{dx}=-2x+2.$ At (3,$^-$4) $\frac{dy}{dx}=\frac{^-(3-1)}{^-4+2}=1$ at $x=3$ $\Rightarrow$ gradient of normal at (3,-4) is -1 $\frac{dy}{dx}=-4$. $\Rightarrow\frac{y-(-4)}{x-3}=-1$ Grad of normal $=\frac{1}{4}$ $y+4=-x+3$ Eqn $\frac{y+4}{x-3}=\frac{1}{4}$. $x+y+1=0$ $4y-x+19=0$ $y=\frac{x}{4}-\frac{19}{4}$.	M_1 for differentiation M_1 for gradient of curve A_1 for gradient of normal. M_1 for equating gradient of normal. A_1 for correct equation of normal **05**
Q8	From $\left(y^2-x^2\right)^2$ $\Rightarrow\left[(\tan\theta+\sin\theta)^2-(\tan\theta-\sin\theta)^2\right]$ $\left[\tan^2\theta+\sin^2\theta+2\sin\theta\tan\theta-\tan^2\theta-\sin^2\theta+2\sin\theta\tan\theta\right]$ $[4\tan\theta\sin\theta]^2$ $=16\tan^2\theta\sin^2\theta$ $=16\tan^2\theta\left(1-\cos^2\theta\right)$	M_1 for expanding and substituting A_1 for simplest form (Simplifying) B_1 for replacing $\sin^2\theta$ with $(1-\cos^2\theta)$

	$=16\left[\tan^2\theta - \sin^2\theta\right]$ $=16\,[\tan\theta - \sin\theta][\tan\theta + \sin\theta]$ $=16\,(x)(y)$ $=16\,xy$ Since L.H.S = R.H.S ,hence $\left(y^2 - x^2\right)^2 = 16xy$ is locus of Q	M_1 for replacing both ($\tan\theta - \sin\theta$) and $\tan\theta + \sin\theta$ with x and y A_1 for correct locus of Q. **05** with a conclusion
Q9	$x = at^2 \text{ and } y = 2at$ $\frac{dx}{dt} = 2at \qquad \frac{dy}{dt} = 2a$ $\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$ $\Rightarrow \frac{y - 2at}{x - at^2} = \frac{1}{t}$ $ty - 2at^2 = x - at^2$ $ty = x + at^2$ is eqn of tangent At P, y =0 $\Rightarrow$ $0 = x + at^2 \Rightarrow x = -at^2$ $P\left(-at^2, 0\right)$ where tangent meets x-axis Q(-a , 2at) where straight line meets the directix. S (a ,o) as being focus of the parabola.  x= -a direcctrix QS is diagonal as well as TP Gradient QS $= \frac{2at - 0}{-a - a} = -t$. Grad TP which tangent is $\frac{1}{t}$ product of grad of GS and TP $= -t \times \frac{1}{t} = -1$ hence diagonals meets at 90^0	 M_1 for gradient of the curve at T which is equal to gradient of tangent at T M_1 for equating gradient of tangent at T A_1 for equation of tangent. B_1 for point P B_1 for point Q B_1 for point S M_1 for getting gradient of diagonals. A_1 for conclusion on product of gradients of diagonals.

	$\|QS\| = \sqrt{(a+a)^2+(0-2at)^2}$ $\sqrt{4a^2+4a^2t^2}$ $2a\sqrt{(1+t^2)}$ $\|TP\| = \sqrt{(at^2+at^2)^2+(2at-0)^2}$ $\sqrt{4a^2t^4+4a^2t^2}$ $\sqrt{4a^2t^2(t^2+1)}$ $2at\sqrt{(t^2+1)}$ $\|QS\| \neq \|TP\|$ length of diagonals are not equal. Grad of $\overline{PQ} = \frac{2at - 0}{at^2 - a} = \frac{2at}{a(t^2-1)} = \frac{2t}{t^2 - 1}$ Grad of $\overline{ST} = \frac{2at - 0}{at^2 - a} = \frac{2t}{t^2-1}$ $\therefore$ grad of $\overline{PQ}$=grad of $\overline{ST}$ grad of $\overline{QT} = \frac{2at - 2at}{at^2 - a} = 0$ grad of $\overline{PS} = \frac{0 - 0}{a+at^2} = 0$ $\therefore$ Grad of QT = grad of $\overline{PS}$, hence opposite sides are parallel to each other. $\|\overline{TS}\| = \sqrt{(a - at^2)^2+(2at - 0)^2}$ $\sqrt{a^2(1-t^2)^2+4a^2t^2} = a\sqrt{4t^2+(1-t^2)^2}$ $a\sqrt{(4t^2+t^4-2t^2+1)}$ $a\sqrt{(t^4+2at+1)}$ $a(t^2+1)$ $\|\overline{TS}\| = \sqrt{(a - at^2)^2+(2at - 0)^2} = a(t^2+1)$ $\|\overline{QP}\| = \sqrt{(a+at^2)^2+(-2at)^2} = a(t^2+1)$ $\|\overline{PS}\| = \sqrt{(a+at^2)^2+0^2} = a(t^2+1)$ Since $\|TS\| = \|\overline{TQ}\| = \|\overline{QP}\| = \|PS\| = a(t^2+1)$ hence sides are equal. Hence STQP is rhombus	M₁ for getting lengths of diagonals A₁ conclusion on lengths of diagonal. M₁ for getting gradient of opposite sides. A₁ for conclusion on gradients of opposite sides. M₁ for getting lengths of all sides. A₁ for conclusion of length of all sides max of 3 properties of rhombus for 06 **12**
Q10 (a)	from $A = \frac{PR(R^n - 1)}{R-1}$ given P = 800,000 R = (1 + r) = 1.05 n= 4 number of times interest has been calculated.	B₁ for value of P. B₁ for value of R. B₁ for value of n.

	$\Rightarrow A = 800{,}000(1.05)\frac{[1.05^4 - 1]}{1.05-1}$	M_1 for substituting for P,R and n
		M_1 for simplifying
	$=3{,}620{,}505/=$	A_1 for correct amount receive at end of 2022
(b)	$(1+x)^{\frac{1}{3}} = 1 + \frac{1}{3}x + \frac{1}{3}\left(\frac{1}{3}-1\right)\frac{X^2}{21} + \ldots\ldots\ldots\ldots$ $= 1 + \frac{x}{3} - \frac{x^2}{9}$	B_1 for expansion of $(1+3x)^{\frac{1}{3}}$
	$(1+3x)^{\frac{-1}{3}} = 1 + \frac{-1}{3}(3x) + \frac{-1}{3}\left(\frac{-1}{3}-1\right)\frac{(3x)^2}{2} + \ldots\ldots\ldots\ldots$ $= 1 - x + 2x^2$	B_1 for expansion of $(1+3x)^{-\frac{1}{3}}$
	$\left(\frac{1+x}{1+3x}\right)^{\frac{1}{3}} = \left(1+\frac{x}{3}-\frac{x^2}{9}\right)(1-x+2x^2)$	M_1 for substituting in $(1+x)^{\frac{1}{3}}$ $(1+3x)^{\frac{1}{3}}$
	$= 1 - x + 2x^2 + \frac{x}{3} - \frac{x^2}{3} - \frac{x^2}{9} + \ldots\ldots\ldots\ldots$ $= 1 - \frac{2x}{3} + \frac{14x^2}{9}$	A_1 for correct expansion of $\left(\frac{1+x}{1+3x}\right)^{\frac{1}{3}}$
	$\left(\frac{1+\frac{1}{125}}{1+\frac{3}{125}}\right)^{\frac{1}{3}} = 1 - \frac{2}{3}\left(\frac{1}{125}\right) + \frac{14}{9}\left(\frac{1}{125}\right)^2$	M_1 for substituting for x
	$\left(\frac{126}{128}\right)^{\frac{1}{3}} = \left(\frac{63}{64}\right)^{\frac{1}{3}} = \frac{\sqrt[3]{63}}{4}$ $\Rightarrow \sqrt[3]{63} = 4\left(1 - \frac{2}{375} + \frac{14}{140625}\right)$ $\therefore \sqrt[3]{63} = 3.9791$	A_1 for cube root of 63. **06**
Q11.	$\frac{3+2x+x^2}{x^3(x+2)} = A/x + B/x^2 + C/X^3 + D/x+2$ $3+2x+x^2 = Ax^2(x+2) + Bx(x+2) + C(x+2) + Dx^3$	M_1 Splitting into partial frac
	When $x = 0$ $3 = 2C \Rightarrow C = \frac{3}{2}$	M_1 (obtaining A_1
	When $x = -2$ $3 + 2(-2) + (-2)^2 = D(-2)^3$ $3 = -8D \Rightarrow D = \frac{-3}{8}$	~~B~~ A1
	Comparing coffients. $x^3: O = A + D \Rightarrow A = \frac{3}{8}$	~~B~~ A1
	$x^2: 1 = 2A + B$ $1 = 2\left(3/8\right) + B$	

$B = \frac{1}{4}$ — B1

$\therefore \frac{x^2+2x+3}{x^3(x+2)} = \frac{3}{8x} + \frac{1}{4x^2} + \frac{3}{2x^3} - \frac{3}{8(x+2)}$ — B1 (substitution)

$\int \frac{x^2+2x+3}{x^3(x+2)}dx = \frac{3}{8}\int \frac{1}{x}dx + \frac{1}{4}\int x^{-2}dx$ — M1 (Integrating)

$+\frac{3}{2}\int x^{-3}dx - \frac{3}{8}\int \frac{1}{x+2}dx$ — A1

$\frac{3}{8}\ln x - \frac{3}{8}\ln(x+2)$

$\frac{-1}{4x} - \frac{3}{4x^2} + C$ — AQ

$\int_2^5 \frac{x^2+2x+3}{x^3(x+2)}dx = \frac{3}{8}\ln\left[\frac{x}{x+2}\right]_2^5 - \frac{1}{4}\left[\frac{1}{x}\right]_2^5$ — B1 Writing limits & removing C

$\frac{-3}{4}\left[\frac{1}{x^2}\right]_2^5$

$\frac{3}{8}\ln\left(\frac{5}{7}\right) - \frac{3}{8}\ln\left(\frac{2}{4}\right) - \frac{1}{4}\left(\frac{1}{5} - \frac{1}{2}\right)$ — M1 for substituting limits.

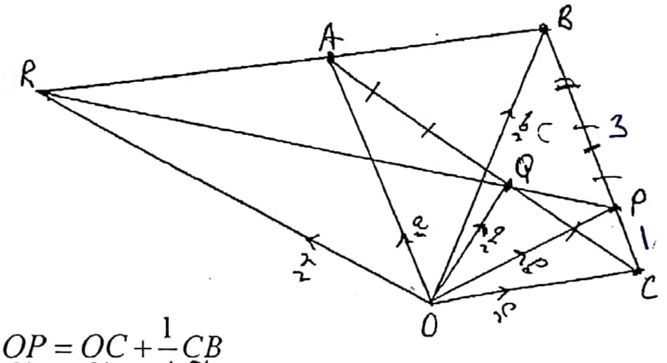
$-\frac{3}{4}\left[\frac{1}{25} - \frac{1}{4}\right]$

$= -0.126177 + 0.259930 + 0.075 + 0.1575$

$= 0.366253 \sim 0.3663.$ — A1 for correct value of integral

12

Q12 (a)



$\underset{\sim}{OP} = \underset{\sim}{OC} + \frac{1}{4}\underset{\sim}{CB}$ — M1 for method

$= \underset{\sim}{OC} + \frac{1}{4}(\underset{\sim}{OB} - \underset{\sim}{OC})$ — ~~A1 for OP~~

$= \underset{\sim}{OC} - \frac{1}{4}(\underset{\sim}{OC} - \underset{\sim}{OB})$

$= \frac{1}{4}(3\underset{\sim}{c} + \underset{\sim}{b})$ — A1 for OP.

M1 for method.

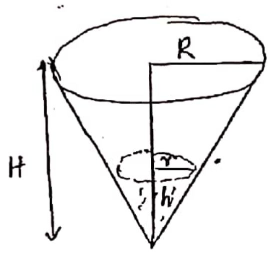
$OQ = OA + \frac{3}{5}AC$

$= O\underset{\sim}{A} + \frac{3}{5}OC - \frac{3}{5}\underset{\sim}{OA}$

$= \frac{1}{5}(2O\underset{\sim}{A} + 3\underset{\sim}{OC})$ — A1 for OQ

$= \frac{1}{5}(2a + 3c)$

	$PB = \frac{3}{4}(CB) \quad = \frac{3}{4}(OB - OC)$	
	$= \frac{3}{4}(OB - OC)$	
	$= \frac{3}{4}(b - c)$	
	Let $BR = t\,BA$	
	$PR = s\,PQ$	
	$PB = PR + RB$	
	$= \cancel{SPQ - SPQ} = sPQ - tBA.$	
	$\frac{3}{4}(b - c) = S(\frac{2}{5}a - \frac{3}{20}c - \frac{1}{4}b) - t(a - b)$	M_1 for combining routes which involved PR BR and PB.
	$\frac{3}{4}b - \frac{-3}{4}c = \left(\frac{2}{5}s - t\right)a + \left(t - \frac{1}{4}s\right)b - \frac{-3s}{20}c$	
	From c direction	
	$-\frac{3}{4} = -\frac{3}{20}s$	
	$S = 5$	$\cancel{B_1}$ A_1
	$QR = QP + PR$	
	$QP + SPR$	
	$\frac{1}{4}(3c + b) + 5\left(\frac{2}{5}a - \frac{-3}{20}c - \frac{1}{4}b\right)$	M_1
	$\frac{3}{4}c + \frac{1}{4}b + 2a - \frac{3}{4}c - \frac{5}{4}b$	A_1 for $QR = 2a - b$
	$= 2a - b$	**08**
(b)	$r = \begin{pmatrix}1\\0\\-2\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\4\end{pmatrix}$	B_1
	$x = 1 + 2\lambda$	
	$y = 0 + 3\lambda$	
	$Z = -2 + 4\lambda$	
	$\Rightarrow 4(1+2\lambda) + 3(0+3\lambda) + 2(-2+4\lambda) = 25$	M_1
	$25\lambda = 25$	A_1
	$\lambda = 1$	
	$\Rightarrow x = 1 + 2(1) = 3$	
	$y = 0 + 3(1) = 3$	
	$z = -2 + 4(1) = 2$	
	Hence they intersect at (3,3,2)	A_1
		04

Q13 (a)	$\frac{dy}{dt} = \sin 3t\,(-2be^{-2t}) + 3be^{-2t}\cos 3t$	M_1 for differentiation.
	$= {}^{-}2\left(be^{-2t}\right)\sin 3t + 3be^{-2t}\cos 3t$	
	$= {}^{-}2y + 3be^{-2t}\cos 3t$	M_1 for replacing $be^{-2t}\sin 3t$
	$\frac{dy}{dt} + 2y = 3be^{-2t}\cos 3t$	B_1 for making $3be^{-2t}\cos 3t$ a subject.
	$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} = -6be^{-2t}\cos 3t - 9be^{-2t}\text{sm}3t$	
	$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} = -2\left(3be^{-2t}\cos 3t\right) - 9(be^{-2t}\sin 3t)$	M_1 for differentiation.
	$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} = -2\left(\frac{dy}{dt} + 2y\right) - 9y$	
	$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} = \frac{2dy}{dt} - 4y - 9y$	M_1 for replacing $3be^{-2t}\cos 3t$ and $be^{-2t}\sin 3t$
	$\therefore \frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 13y = 0$	A_1 for required form.
		06
(b)		
	$\frac{r}{h} = \frac{R}{H} = \frac{5}{15}$	B_1 for expressing r in terms of h
	$r = \frac{h}{3}$	
	$V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$	B_1 for expression of volume.
	$= \frac{\pi}{27}h^3$	
	$\frac{dv}{dh} = \frac{\pi h^2}{9}$	B_1 (differentiating).
	$\frac{dh}{dt} = \frac{dv}{dt} \times \frac{dh}{dv}$	M_1 for substituting $\frac{dv}{dt}$ and $\frac{dh}{dv}$
	$= 2\times\frac{9}{\pi h^2} = \frac{18}{\pi h^2}$	
	$\left.\frac{dh}{dt}\right\vert_{h=5} = 2\times\frac{9}{\pi(25)}$	M_1 for substituting value of h
	$= \frac{18}{25\pi}$ or $0.2292\ \text{cmS}^{-1}$	A_1 for required volume.
		06
Q14 (a)	$\frac{3\sin\theta + \sin 2\theta}{1 + \cos 2\theta + 3\cos\theta}$	
	but $\sin 2\theta = 2\sin\theta\cos\theta$	
	$1 + \cos 2\theta = 2\cos^2\theta$	
	$= \frac{3\sin\theta + 2\sin\theta\cos\theta}{2\cos^2\theta + 3\cos\theta}$	M_1 for converting double angle to single angle.

	$= \dfrac{\sin\theta(3+2\cos\theta)}{\cos(2\cos+3)}$ $= \tan\theta$	M_1 for factorization. A_1 for required prove
	$\Rightarrow \tan\theta + \dfrac{1}{\cos^2\theta} = 2$ $\tan\theta + \sec^2\theta = 2$ $\tan\theta + 1 + \tan^2\theta = 2$ $\tan^2\theta + \tan^2\theta - 1 = 0$ let $\tan\theta$ be t $\Rightarrow t^2 + t - 1 = 0$ $t = \dfrac{-1 \pm \sqrt{1^2 - 4 \times 1 \times -1}}{2 \times 1}$ $= \dfrac{-1 \pm \sqrt{5}}{2}$ $= 0.6180$ or -1.6180 $\Rightarrow \tan\theta = 0.6180$ or $\tan\theta = -1.6180$ $\theta = \tan^{-1}(0.6180)$ or $\theta = \tan^{-1}(-1.6180)$ $\theta = 31.72^0, 211.72^0$ or $121.72^0, 301.72^0$ 04	B_1 for replacing $\dfrac{3\sin\theta+\sin2\theta}{1+3\cos\theta+\cos2\theta}$ with $\tan\theta$ If not corrected, transfer 2mks ↓. M_1 for method of solving quadratic equation. A_1 for correct 4 values of Q **06**
(b)	$7\cos A + 24\sin\theta = R\sin(A+B)$ $R\cos B \sin A + R\cos A \sin B.$ $R\sin B = 7$ ____ (i) $R\cos B = 24$ $R^2 = 7^2 + 24^2 \Rightarrow R = 25$ $\tan B = \dfrac{7}{24}$ $B = 16.26^0$ $\therefore 7\cos A + 24\sin A = \sin(A + 16.26^0)$	M_1 for $R = 25$ without negative on it M_1 for value $\beta = 16.26^0$ A_1 for expressing in terms of $R\sin(A+\beta)$
	For minimum value of $25\sin(A+16.26^0)$ to occur $\sin(A+16.26^0) = {}^-1$ $\therefore \dfrac{2}{2\cos A + 24\sin A + 10} = \dfrac{2}{25(-1)+10} = \dfrac{{}^-2}{15}$	β_1 for value $\dfrac{{}^-2}{15}$
	For maximum value of $25\sin(A+16.26^0)$ t occur $\sin(A+16.26^0) = 1$ $\therefore \dfrac{2}{7\cos A + 24\sin A + 10} = \dfrac{2}{25(1)+10} = \dfrac{2}{35}$	β_1 for value $\dfrac{2}{35}$
	Therefore $\dfrac{2}{7\cos A + 24\sin}$ ranges 08 From $\dfrac{{}^-2}{15}$ to $\dfrac{2}{35}$ OR $\dfrac{{}^-2}{15} \le \dfrac{2}{7\cos + 24\sin A + 10} \le \dfrac{2}{35}$	A_1 for required range. **06**

Q15 **(a)**	$\frac{2\log_2^x}{2}+\log_2(x+6)=\frac{6}{3}\log_2(x+2)$ $\log_2^x+\log_2(x+6)=2\log_2(x+2)$ $\log_2^x x(x+6)=\log_2(x+2)^2$ $\Rightarrow x(x+6)=(x+2)^2$ $x^2+6x=x^2+4x+4$ $2x=4$ $x=2$	M_1 change to same base M_1 for law of Log applied. B_1 for removing log M_1 for expanding and simplifying. A_1 for correct value of x. 05
(b)	$Z=\frac{(3-i)(5+12i)}{(1+3i)^2}$ $=\frac{15+36i-5i+12}{1+3i+3i-9}$ $=\frac{27+31i}{^-8+6i}$ $\frac{27+31i}{(^-8+6i)}\frac{(-8-6i)}{(-8-6i)}$ $\frac{^-216-162i-248i+186}{64+36}$ $=\frac{^-30-410i}{100}$ $Z=\frac{^-3}{10}-\frac{41i}{10}$ (i) $\lvert Z\rvert=\frac{1}{10}\sqrt{9+1681}=4.1110$ units (ii) $\text{Arg } Z=\tan^{-1}\left(\frac{^-41}{^-3}\right)=180^0+\tan^{-1}\frac{41}{3}$ $=266.82^0$ (iii) $Z=4.1110(\cos 266.82^0+i\sin 266.82^0)$	B_1 for simple form of Z. M_1 for realization of Conjugate A_1 for Z in form a+bi B_1 for $\lvert Z\rvert$ correct value. M_1 for method. A_1 for correct Arg Z B_1 for polar form. **07**
Q16	(i) Let number of students that are already voted be P $\frac{dp}{dt}=Kp(1405-P)$ $\frac{dp}{p(1405-p)}=Kdt$ but $\frac{1}{P(1405-p)}=\frac{1}{1405p}+\frac{1}{1405(-1405-p)}$ $\frac{1}{1405}\int\frac{1}{p}dp+\frac{1}{1405}\int\frac{1}{1405-p}dp=\int Kdt$	B_1 for D.E formed M_1 for separating variables. M_1 for integrating.

	$\frac{1}{1405}\ln\left(\frac{P}{1405p}\right) = Kt + C$	A_1 for correct integral.
	When t= 0 and P = 20 $\frac{1}{1405}\ln\frac{20}{1385} = K(o) + C$ $C = \frac{1}{1405}\ln\left(\frac{20}{1385}\right)$	(Initial Condition) M_1 for substituting t = 0 and p = 20. A_1 for value of constant of integration.
	$\Rightarrow \frac{1}{1405}\ln\left(\frac{P}{1405\text{-}p}\right) = Kt + \frac{\ln}{1405}\left(\frac{20}{1385}\right)$ $\frac{1}{1405}\ln\left(\frac{P}{(1405\text{-}p)} \times \frac{1385}{20}\right) = Kt$	
	When t = 3, p = 600 $\frac{1}{1405}\ln\left(\frac{600}{805} \times \frac{1385}{20}\right) = 3K$	M_1 for substituting t = 3 and p= 600
	$K = \frac{1}{1405}\ln\left(\frac{600}{805} \times \frac{1385}{20}\right) = 9.3566086 \times 10^{-4}$	A_1 for value of K
	$\frac{1}{1405}\ln\left(\frac{P}{1405\text{-}p} \times \frac{1385}{20}\right) = 9.3566086 \times 10^{-4}t$	
	When t = 1 and p = ? $\frac{1}{1405}\ln\left(\frac{P}{1405\text{-}p} \times \frac{1385}{20}\right) = 9.3566086 \times 10^{-4}(1)$	M_1 for substituting t = 1
	$\ln\left(\frac{P}{1405\text{-}p} \times \frac{1385}{20}\right) = 1.314603507$ $\frac{P}{(1405\text{-}p)} \times \frac{1385}{20} = e^{1.314603507}$ $= 3.7232744$ $\frac{P}{1405} = 3.7232744 \times \frac{20}{1385}$ $= 0.053765695$ $1.053765695\,p = 75.54080272$ $P = 71.6853$ $\cong 71$ students	A_1 for correct numbers of students. M_1 for substituting P = 800
(ii)	When P = 800 and t = ? $\Rightarrow \frac{1}{1405}\ln\left(\frac{800}{605} \times \frac{1385}{20}\right) = 9.356608595 \times 10^{-4}t$ T = 3.44 hours hence at 10 : 26 a.m	A_1 for correct time required. **12**

END